Calculate The Uncertainty Of NF Measurements

Simple modifications to the basic noise-figure equations can help in predicting uncertainties associated with test equipment.

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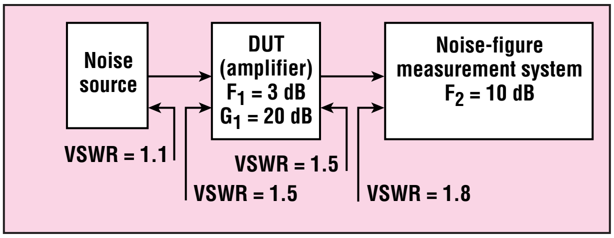
RAPID growth in satellite-communications and mobile-communications markets has increased the demand for monolithic and discrete semiconductor devices with lower noise figures (NFs). With these low-noise devices comes the pressure to reduce the NF measurement uncertainty. What follows is a model for calculating the uncertainty of NF measurements, along with an easy-to-follow example.

It is often assumed that the uncertainty due to mismatch is the largest source of NF measurement uncertainty, although this is rarely the case. In fact, the uncertainty of the measurement system's NF, the uncertainty related to the noise source, the NF and architecture of the measurement system, as well as the gain of the device under test (DUT) have a significant bearing on the overall measurement uncertainty. If any of these parameters are unfavorable, the uncertainty due to mismatch will have little impact on the overall result. Engineers can waste time and money using network analysis to perform S-parameter correction of mismatches. Correction introduces a number of other issues and in reality greater attention to the other parameters in the system can yield more significant improvements.



1. This measurement system can be used for evaluating the NF of low-noise amplifiers (LNAs) and devices.

The procedures and following example that will be shown are designed to increase an engineer's familiarity and understanding of NF measurement uncertainty calculations. A programmed example offers an easy-to-follow model for calculating the uncertainty in any measurement configuration. The model and some further examples clearly show which parameters have the most-significant impact on the uncertainty. A discussion of the main parameters includes the benefits to be gained by improving particular parameters of a measurement system. In addition, a spreadsheet is available upon request from the author (at the e-mail address listed) to automate the uncertainty calculations.

Table 1: Log to linear transformations for noise figure and gain

Parameter	Log value	Linear = 10^(dB/10)
F_1	3 dB	1.995
F_2	10 dB	10
G_1	20 dB	100
$F_{12} = F_1 + (F_2 – 1)/G_1$	3.19 dB	2.085

The uncertainty of NF measure-

Table 2: Calculating noise-figure and gain ratios figure

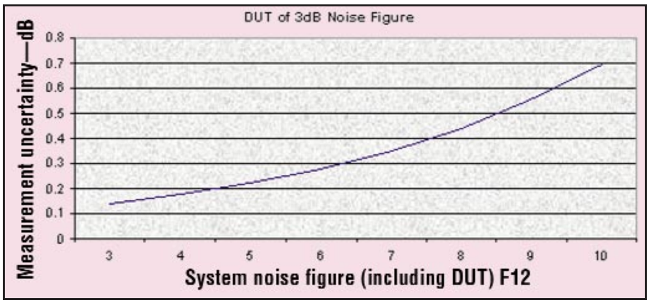
Parameter	Ratio
F_{12}/F_1	1.045
F_2/F_1G_1	0.050
$(F_2 - 1)/F_1G_1$	0.045
$F_{12}/F_{1)} - (F_2/F_1G_1)$	0.995

ments can be calculated by eq. 7 from the sidebar. One application where NF is important is during the testing of amplifiers. A typical setup would include a noise source, the DUT, as well as an NF measurement system (Fig. 1).

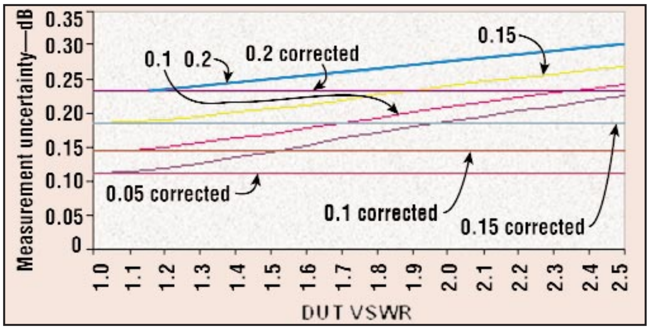
The model provides the measurement uncertainty associated with a particular NF based on the knowledge of individual VSWRs within the system and the specifications of the measurement system. For the model to be accurate, the DUT must have reasonable reverse isolation, so that a mismatch on one port does not drastically affect the impedance seen at the other port.

The first step in using the model requires the operator to refer to the test-equipment owner's manuals to calibrate the NF receiver, apply the DUT, then record the DUT's corrected gain (G_1) and NF (F_1).

Then the NF receiver's autoranging function must be switched off so that the attenuators remain in the same



2. The NF measurement uncertainty increases with increasing test-system measurement uncertainty.



3. Instruments or systems with better NF uncertainty generally outperform systems employing S-parameter correction.

position. Following this, the DUT is removed and the uncorrected NF of the test receiver (F_2) is measured. All of the decibel values must be converted into equivalent linear values and the NF value of F_{12} must be calculated according to Table 1. A series of ratios are then calculated according to Table 2. The various VSWR values can then be converted into reflection coefficients through Table 3. The resulting reflection coefficients can then be used to calculate the various impedance-matching uncertainties of the NF measure-

DERIVING THE NF UNCERTAINTY EQUATION

The general equation for the noise figure (NF) of two cascaded stages is:

$$F_{12} = F_1 + \frac{F_2 - 1}{G_1} \qquad (1)$$

where:

F_1 = the linear NF of the DUT,

F_2 = the linear NF of the NF measurement receiver,

F_{12} = the linear NF of the complete system (DUT and measurement receiver), and

G_1 = the linear gain of the DUT.

Since it is the uncertainty of the DUT's NF (F_1) that is of interest, the terms can be rearranged by:

$$F_1 = F_{12} - \left(\frac{F_2 - 1}{G_1}\right) \qquad (2)$$

Because F_1 is dependent on the three independent variables F_{12}, F_2, and G_1, differential calculus in the form of Taylor's Theorem can be applied to find the uncertainty of F_1:

$$\delta F_1 = \frac{\partial F_1}{\partial F_{12}}\delta F_{12} + \frac{\partial F_1}{\partial F_2}\delta F_2 + \frac{\partial F_1}{\partial G_1}\delta G_1 \qquad (3)$$

where:

δF_1 = the uncertainty of the DUT NF,

δF_2 = the uncertainty of the measurement receiver's NF,

δF_{12} = the uncertainty of the complete system (the DUT and measurement receiver) NF, and

δG_1 = the uncertainty of the DUT gain.

From eq. 2:

$$\frac{\partial F_1}{\partial F_{12}} = 1$$

$$\frac{\partial F_1}{\partial F_2)} = -\left(\frac{1}{G_1}\right)$$

$$\frac{\partial F_1}{\partial G_1} = \frac{F_2 - 1}{G_1^2} \qquad (3a)$$

so that:

$$\delta F_1 = \delta F_{12} - \left(\frac{1}{G_1}\right)\delta F_2 + \left(\frac{F_2 - 1}{G_1^2}\right)\delta G_1 \qquad (4)$$

RF engineers generally work in logarithmic decibels, so using $NF_1(dB) = 10\log F_1$ and the standard differential coefficient:

$$Log_a \chi = \frac{1}{\chi Lna} \qquad (4a)$$

results in:

$$\delta NF_1 = \frac{4.34}{F_1}\delta F_1 \qquad (4b)$$

Applying the same procedure for δF_{12}, δF_2, and δG_1, results in:

$$\delta F_{12} = \frac{\delta NF_{12} F_{12}}{4.34}$$

$$\delta F_2 = \frac{\delta NF_2 F_2}{4.34} \quad \delta G_1 = \frac{\delta G_1(dB) G_1}{4.34} \qquad (4c)$$

Substituting these into eq. 4 and simplifying yields:

$$\delta NF_1 = \left(\frac{F_{12}}{F_1}\right)\delta NF_1 - \left(\frac{F_2}{F_1 G_1}\right)\delta NF_2 + \left(\frac{F_2 - 1}{F_1 G_1}\right)\delta G_1\,(dB) \qquad (5)$$

The three δ terms in the previous equation are due to the NF measurement receiver and the DUT. However, NF instruments rely on a calibrated noise source with a specified excess noise ratio (ENR). Clearly, there will be an uncertainty associated with this ENR and this will contribute to the overall uncertainty equation.

When the DUT is a frequency converting device with F_2 and F_{12} at different frequencies, δENR is included as part of δNF_{12}, δNF_2, and δG_1. However, when the DUT is an amplifier with F_2 and F_{12} being measured at the same frequency, the effect on δG_1 cancels out. The δENR then only influences the first two terms in eq. 5. This produces a fourth term derived from eq. 5 as follows:

$$\left(\frac{F_{12}}{F_1} - \frac{F_2}{F_1 G_1}\right)\delta ENR \qquad (6)$$

where:

δENR = the uncertainty of the noise source's ENR.

This term should only be included when measuring amplifiers.

Since the causes of the uncertainties in the four δ factors are different, the terms can be combined in a root-sum-of-squares (RSS) fashion, which provides a realistic overall uncertainty value.

The equation for the overall NF uncertainty is therefore:

$$\delta NF = \left\{\left[\left(\frac{F_{12}}{F_1}\right)\delta NF_{12}\right]^2 + \left[\left(\frac{F_2}{F_1 G_1}\right)\delta NF_2\right]^2 + \left[\left(\frac{F_2 - 1}{F_1 G_1}\right)\delta G_1(dB)\right]^2 + \left[\left(\frac{F_{12}}{F_1} - \frac{F_2}{F_1 G_1}\right)\delta ENR\right]^2\right\}^{0.5} \qquad (7)$$

Equation 7 provides the measurement uncertainty associated with a particular NF, using a system's VSWR characteristics and general electrical specifications. By knowing the NF uncertainty, greater confidence can be placed in the measured specifications attributed to production amplifiers and active devices.

Table 3: VSWR-to-reflection-coefficient transformations

	VSWR	Reflection coefficient ρ = (VSWR - 1)/(VSWR + 1)
Noise source	1.10:1	0.048
DUT input	1.50:1	0.20
DUT output	1.50:1	0.20
Instrument	1.80:1	0.286

ment system (Table 4).

The next step involves the calculation of the overall uncertainties using the maximum matching uncertainties and the NF instrument uncertainties. The instrumentation uncertainties should be those specified by the manufacturer. For this example, assume an instrument NF uncertainty (δInstrumentNF) of 0.05 dB, based on instrument-gain uncertainty (δInstrumentGain) of 0.15 dB, and effective-noise-ratio (ENR) uncertainty (δENR) of 0.1 dB. The following calculations show how to calculate the various NF, gain, and ENR uncertainties, although it should be noted that the receiver-only uncertainty (δENR_{RXOnly}) is not used in this example since the DUT is an amplifier:

$$\delta NF_{12}(dB) = [(\delta_{NS-DUT})^2 + (\delta InstrumentNF)^2 + (\delta ENR_{RXOnly})^2]^{0.5} = \sqrt{0.083^2 + 0.05^2} = \underline{0.097} \qquad (1)$$

$$\delta NF_2(dB) = [(\delta_{NS-NFI})^2 + (\delta InstrumentNF)^2 + (\delta ENR_{RXOnly})^2]^{0.5} = \sqrt{0.119^2 + 0.05^2} = \underline{0.129} \qquad (2)$$

$$\delta G_1(dB) = [(\delta_{NS-DUT})^2 + (\delta_{NS-NFI})^2 + (\delta_{DUT-NFI})^2 + (\delta InstrumentGain)^2 + (\delta ENR_{RXOnly})^2]^{0.5} = [(0.083^2 + 0.119^2 + 0.511^2 + 0.15^2]^{0.5} = \underline{0.552} \qquad (3)$$

$$\delta ENR(dB) = \underline{0.1} \qquad (4)$$

The next step involves the calculation of uncertainty terms (shown in Table 5) through multiplying the ratios found in Table 2 by the appropriate uncertainties.

There are many ways of calculating the overall uncertainty of a measurement. The traditional root-sum-of-squares (RSS) method will be used as the final step in this example since it is well-understood. RSS should, of course, use linear quantities, but with decibel values of the order that are dealt with here, the error is approximately 0.001 dB. Overall RSS measurement uncertainty is then:

$$RSS\ measurement\ uncertainty = \pm(0.102^2 + 0.007^2 + 0.025^2 + 0.099^2)^{0.5} = \pm 0.144 \qquad (5)$$

The NF of the DUT in this example is therefore 3 ± 0.144 dB. The results of the calculations for Table 5 indicate that parameters $\pm\delta NF_{12}$, F_{12}/F_1, and δENR have the most significant influence on measurement uncertainty. These parameters as well as one other factor not appearing in these equations will now be explored in detail.

One of the most significant parameters affecting the uncertainties in Table 5 is δENR, the uncertainty of the noise source. For best uncertainty when measuring low-noise devices, low ENR sources should be used. This results in a lower δInstrumentNF since the low ENR exercises less of the measurement detector's dynamic range. There is a further advantage to using a low-ENR source in that its impedance is more constant between the on and off states. This is because a low ENR source (with ENR of typically 5 dB) is basically a high-ENR source (with ENR of typically 15 dB) with an additional attenuator. Beyond these points, there is not much room for movement with the noise source, since δENR is referred to the National Institute of Standards and Technology (NIST).

The instrument architecture includes any frequency translations that enable measurements at reasonable intermediate frequencies (IFs). The measurement-receiver architecture is either a single-sideband (SSB) or double-sideband (DSB) architecture. Network-analyzer-based instruments use the DSB architecture. This being the case, there is the possibility that power will appear in the undesired sideband causing a measurement error.

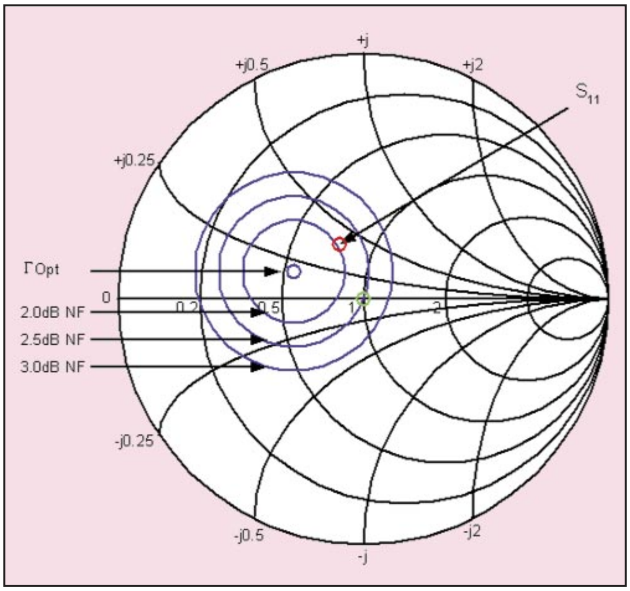
The possibility of this error can be reduced by using a narrowband DSB architecture. However, this increases the measurement time dramatically. For example, the theoretical measurement time increase when going from a 4-MHz bandwidth to a 40-kHz bandwidth is 100 times. Instruments employing a SSB structure do not have a problem with uncertainty due to power being in the unwanted sideband since it is filtered out.

The ratio of system NF [(DUT and measurement instrument)/DUT NF, F_{12}/F_1] was also shown to have a sig-

Table 4: Calculating impedance-matching uncertainties

Ports source – load	Negative uncertainty 1 = -20 log (1 – ρsourceρload)	Positive uncertainty = 20 log (1 + ρsourceρload)	Maximum uncertainty (dB)
Noise source – DUT $_{INPUT}$	0.083	0.082	0.083
Noise source – instrument	0.119	0.117	0.119
DUT $_{output}$ – Instrument	0.511	0.483	0.511

nificant effect on the overall uncertainty. Parameter F_{12} is a function of the instrument NF and the DUT gain and NF. Figure 2 shows how the measurement uncertainty increases with an increase in F_{12}. The data are based on the previous example. In this case, an F_{12} value of 3 dB is the best that can be achieved since this is the NF of the DUT. Low DUT gain and/or high F_2 increase F_{12} and result in a higher uncertainty. Most combinations of instruments and DUTs with gain will produce results in the left-hand portion of the graph. DUTs with negative gain, such as mixers, move into the right-hand portion of the graph, increasing the uncertainty. In this situation, the uncertainty can be improved by inserting a low-noise amplifier (LNA) between the DUT and the measurement receiver.



4. Conjugate matching does affect the NF measured for a DUT. The center of the chart indicated a characteristic system impedance of 50 {CAP OMEGA}.

Parameter δNF_{12} is made up of δInstrumentNF and the mismatch uncertainty. Work can be performed on the mismatch uncertainty using S-parameter correction but δInstrumentNF is a fixed function of the instruments linearity. Table 6 shows the uncertainty of the example DUT and some further typical DUTs against different δInstrumentNF values. The increase in the uncertain-


5. The HP 8970B NF meter has long been a precision tool for simultaneously characterizing the NF and gain of low-noise devices.

Table 5: Calculating uncertainty terms

	Result (dB)
$(F_{12}/F_1) \times \delta NF_{12}$	0.102
$(F_2/F_1G_1) \times \delta NF_2$	0.007
$[(F_2 - 1)/(F_1G_1)] \times \delta G_1$	0.025
$[(F_{12}/F_1) - (F_2/F_1G_1)] \times \delta ENR$	0.099

ty with increase in δInstrumentNF is apparent. It is also clear from this data that an instrument with good δNF and no correction can outperform an instrument with idealized S-parameter correction but slightly worse δNF.

Figure 3 shows the change in measurement uncertainty of a typical DUT against an increase in VSWR. The data are shown with and without idealized S-parameter correction. From Table 7, it can be seen that instruments with good δNF and no correction of mismatches outperform other instruments up to particular VSWRs dependent on the δNF.

Depending on the measurement instrument and the VSWR of the DUT, S-parameter correction of mismatches may then appear to be of some benefit. However, there is one fundamental flaw in using S-parameter correction of mismatches when measuring NF. S-parameters provide almost all of the information needed for a device, with one exception—noise.

It is true that although a device is unlikely to be perfectly matched in practice, S-parameters can be used to obtain the available gain (the gain with the input and output conjugately matched). However, it has been cited that if this available gain is used in the following standard equation to remove the effects of the second-stage NF, that a more-accurate result will be achieved:

$$NF_{DUT} = NF_{DUT+Instrument} - \left(\frac{NF_{Instrument} - 1}{Gain_{DUT}}\right) \quad (6)$$

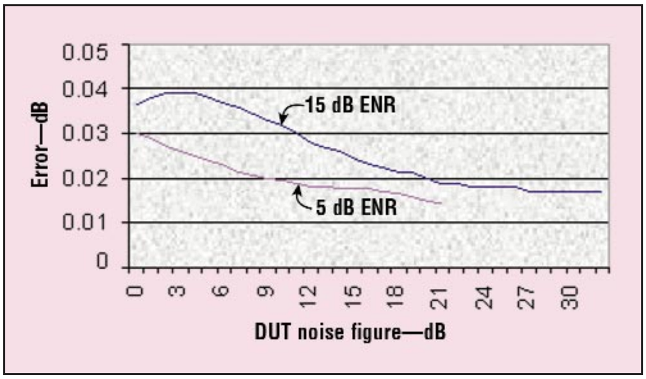
This theory assumes that the NF of the device (NF_{DUT}) and measuring instrument ($NF_{Instrument}$) do not change with impedance. It would be nice if this were the case, but unfortunately NF varies wildly with impedance as a glance at the noise data for any RF device will show. An example is shown below in graphical form in Fig. 4.

An NF measuring instrument will measure the NF and gain with 50 Ω presented to the input of the device (the circle at the center of the chart).

Table 6: Comparing DUT and instrument NF uncertainties

δInstrumentNF	0.05 dB	0.10 dB	0.15 dB	0.20 dB	0.05 dB	0.10 dB	0.15 dB	0.20 dB
	No correction				Idealized S-parameter correction			
DUT (amplifier)	Measurement uncertainty (dB)							
Gain = 20 dB NF = 3 dB Instrument NF = 10 dB In/out VSWR = 1.50:1	±0.144	±0.170	±0.207	±0.249	±0.113	±0.145	±0.186	±0.232
Gain = 13 dB NF = 2.2 dB Instrument NF = 5 dB In/out VSWR = 1.80:1	±0.176	±0.199	±0.232	±0.272	±0.111	±0.145	±0.189	±0.236
Gain = 26 dB NF = 3.5 dB Instrument NF = 10 dB In/out VSWR = 2.0:1	±0.180	±0.200	±0.230	±0.266	±0.112	±0.142	±0.181	±0.225
Gain = 18 dB NF = 0.8 dB Instrument NF = 4 dB In/out VSWR = 2.0:1	±0.181	±0.201	±0.232	±0.268	±0.111	±0.142	±0.182	±0.227

S-parameters could then be used to calculate the available gain, simulating what the gain would be if the input and output of the device were presented with the conjugate of S_{11} (the red circle) and S_{22}, respectively. If the input of the DUT is actually presented with the conjugate of S_{11}, however, its NF would change by approximately 0.5 dB (shown by the blue NF circles in Fig. 4). Any S-parameter correction in NF must therefore be used with great care since it can make the measurements significantly less accurate without knowledge of how the NFs of the DUT and the measurement receiver are altered by conjugate matching.



6. The measurement error of the HP 8970B NF meter is typically less than 0.05 dB.

NOISE PARAMETERS

There are several basic noise parameters that completely describe the noise characteristics of a device. These are the minimum possible NF of the DUT (NF_{min}), the equivalent noise resistance of the device (R_n), and the optimum source-reflection coefficient (Γ_{opt}) for magnitude and phase. These parameters are found by applying different impedances to the device using a tuner for optimization and are completely unrelated to the S-parameters.

There are some other issues that are associated with S-parameter corrections, the first being the need for a network analyzer. This approach to measuring NF is likely to cost several tens of thousands of dollars more than an approach that is without correction. That is not an issue if the area of interest is device characterization, although this will also require bias tees in addition to complex automatic tuner units to present devices with a variety of impedances. NF instruments that are embedded within network analyzers do not then provide a complete solution for device characterization since they can only present the device with a fixed impedance.

NF INSTRUMENTS EMBEDDED WITHIN NETWORK ANALYZERS DO NOT PROVIDE A COMPLETE SOLUTION FOR DEVICE CHARACTERIZATION.

Another problem with using S-parameter correction on NF is the increased time to make the measurement due to the extra calibrations and supplemental measurements. The connecting/disconnecting of cables, bias tees, tuners, and more, also adds reliability issues, measurement uncertainty, and a time penalty.

Considering the number and diversity of operations that are required to make S-parameter-corrected NF measurements, it becomes clear that some form of computer control of all the operations will be required. This adds further cost and complexity. The general complexity of corrected NF measurements/ device characterization suggests that a dedicated test system is required.

MEASURING NF

One current solution that can be used to measure NF is the HP 8970B NF meter from Hewlett-Packard Co. (Fig. 5). Together with an appropriate noise source, the HP 8970B is capable of simultaneously characterizing the NF and gain for receiver systems, their subassemblies, as well as components such as amplifiers, mixers, filters, diplexers, and low-noise block downconverters (LNBs). The low instrumentation uncertainty of the HP 8970B combined with its true SSB receiver architecture and correction to remove the measurement-system noise contribution bring ease and confidence to NF measurements. While the δInstrumentNF for the HP 8970B is specified as 0.1 dB, it is typically much better than this. Typical data from current production HP 8970B units show that the error is typically less than 0.05 dB (Fig. 6) and the advantage of the low ENR source can be clearly seen. ●●

Table 7: Comparing instruments with different NF uncertainties

δInstrumentNF	0.10 dB corrected	0.15 dB corrected	0.20 dB corrected
VSWR where an instrument having δNF = 0.05 dB without mismatch correction provides better measurement uncertainty	≤1.5:1	≤2.0:1	≤2.6:1
VSWR where an instrument having δNF = 0.1 dB without mismatch correction provides better measurement uncertainty	–	≤1.7:1	≤2.4:1
VSWR where an instrument having δNF = 0.15 dB without mismatch correction provides better measurement uncertainty	–	–	≤1.9:1